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CALCULATION OF THE INTENSITY OF ELECTROMAGNETIC FIELDS OF THERMAL MICROWAVE DETECTORS AT HIGH TEMPERATURES. II

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The second moments are found for the spectral amplitudes of the thermal electromagnetic field of a dielectric inhomogeneity of complicated geometry heated to a temperature T.

The second moments of the spectral amplitudes of the thermal field of a dielectric structure with an arbitrary geometry consisting of steps and rods can be determined on the basis of the method of the generalized scattering matrix and [1, 2]. Specifying the temperature dependence $\varepsilon_j(T)$ of the dielectric constant of step j, we extend this method to the solution of analogous problems incorporating a temperature gradient in the inhomogeneities. Choosing as a basic inhomogeneity a dielectric inclusion of finite length, we can reduce the number of calculation procedures to a level about 2^n times lower than that for a semiinfinite step (here n is the number of elements in the structure selected).

1. Dielectric Inhomogeneity of Finite Length in a Waveguide

We seek a solution of the problem of the diffraction of an H_{p0} wave by a dielectric inclusion of bounded length in a rectangular waveguide (Fig. 1a) by the method of [1]. We make use of the symmetry of the inhomogeneity with respect to the plane z = d/2. We divide the incident field into parts of even and odd parity. This problem is reduced to two equivalent problems. The structure of the first problem is shown in Fig. 1b, where there is an electrical wall in the plane z = d/2. By placing a magnetic wall in the same plane, we find the geometry of the second problem. We denote by R^-_{mp} and R^+_{mp} the amplitudes of the harmonics of the waves reflected in region A, which are found through a solution of these two problems. According to the superposition principle, the amplitudes of the wave harmonics reflected from a dielectric inclusion of bounded length are

$$R_{mp} = (R^+_{mp} + R^-_{mp})/2,$$

and the amplitudes of the harmonics of the transmitted waves are

$$T_{mp} = (R^+_{mp} - R^-_{mp})/2.$$

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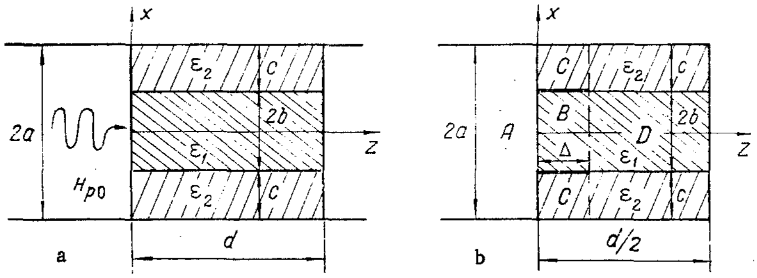

Fig. 1. Stratified dielectric of finite length in a rectangular waveguide. a) Structure under consideration; b) structure equivalent to that under consideration, with the auxiliary geometry.

As in [1], we introduce an auxiliary structure, consisting of an infinitesimally thin, ideally conducting metal strip. Joining the fields, and proceeding by analogy with Part I of [1], we find

$$\begin{gathered}
\sum_{m=1}^{\infty}\left\{R_{ml}\left(\frac{1}{h_{ma}+\Gamma_{qb}}+\frac{\rho_{qb}}{h_{ma}-\Gamma_{qb}}\right)+T_{ml}\left(\frac{\tau_{qb}}{h_{md}-h_{qb}}+\frac{\delta_m\tau_{qb}}{h_{md}+h_{qb}}\right)\right.=A_l\left(\frac{1}{h_{la}-\Gamma_{qb}}+\frac{\rho_{qb}}{h_{la}+\Gamma_{qb}}\right),\\
\sum_{m=1}^{\infty}\left\{R_{ml}\left(\frac{1}{h_{ma}-\Gamma_{qb}}+\frac{\rho_{qb}}{h_{ma}+\Gamma_{qb}}\right)+T_{ml}\left(\frac{\tau_{qb}}{h_{md}+h_{qb}}+\frac{\delta_m\tau_{qb}}{h_{md}-h_{qb}}\right)\right.=A_l\left(\frac{1}{h_{la}+\Gamma_{qb}}+\frac{\rho_{qb}}{h_{la}-\Gamma_{qb}}\right),\\
\sum_{m=1}^{\infty}\left\{R_{ml}\left(\frac{1}{h_{ma}+\Gamma_{qc}}+\frac{\rho_{qc}}{h_{ma}-\Gamma_{qc}}\right)+T_{ml}\left(\frac{\tau_{qc}}{h_{md}-h_{qc}}+\frac{\delta_m\tau_{qc}}{h_{md}+h_{qc}}\right)\right.=A_l\left(\frac{1}{h_{la}-\Gamma_{qc}}+\frac{\rho_{qc}}{h_{la}+\Gamma_{qc}}\right),\\
\sum_{m=1}^{\infty}\left\{R_{ml}\left(\frac{1}{h_{ma}-\Gamma_{qc}}+\frac{\rho_{qc}}{h_{ma}+\Gamma_{qc}}\right)+T_{ml}\left(\frac{\tau_{qc}}{h_{md}+h_{qc}}+\frac{\delta_m\tau_{qc}}{h_{md}-h_{qc}}\right)\right.=A_l\left(\frac{1}{h_{la}+\Gamma_{qc}}+\frac{\rho_{qc}}{h_{la}-\Gamma_{qc}}\right).
\end{gathered}\tag{1}$$

Equations (1) of the present article differ from Eqs. (1) and (3) of [1] in the presence of a term with a coefficient δ_m, which arises due to the satisfaction of the boundary conditions at the $z = d/2$ boundary. For the case of an electrical wall we have $\delta_m = \exp(-ih_{md}d)$, and the matrix R_{ml} should be understood to represent R^{-}_{ml}. In the case of an even-parity excitation, we have $\delta_m = -\exp(-ih_{md}d)$, and R_{ml} gives R^{+}_{ml}.

We solve Eqs. (1) by a small perturbation method. The amplitudes of the harmonics of the reflected field are determined as the residues of the function of the complex variable $h(w)$ at the points h_{ma} ($m = 1, 2, 3, \ldots$). The equation for $h(w)$ is

$$h(w)=\frac{R_0}{\left(1+\dfrac{w}{h_{la}}\right)}\prod_{m=1}^{\infty}\frac{\left(1-\dfrac{w}{h_{mc}}\right)\left(1+\dfrac{w}{h_{mc}}\right)\left(1-\dfrac{w}{\Gamma_{mb}}\right)\left(1+\dfrac{w}{\Gamma_{mb}}\right)}{\left(1-\dfrac{w}{h_{ma}}\right)\left(1+\dfrac{w}{h_{md}}\right)}\times$$
$$\times\left[1+\sum_{m=1}^{M_u-1}U_m\frac{w}{\Gamma_{mb}-w}+\sum_{m=1}^{M_v-1}V_m\frac{w}{\Gamma_{mb}+w}+\tilde U\sum_{m=M_u}^{\infty}\frac{wm^{1-\varkappa}}{\Gamma_{mb}-w}+\tilde V\sum_{m=M_v}^{\infty}\frac{wm^{1-\varkappa}}{\Gamma_{mb}+w}+\sum_{m=1}^{\infty}W_m\frac{w}{h_{md}-w}\right],\tag{2}$$

where $\{U_m\}$, $\{V_m\}$, $\tilde U$, $\tilde V$ are unknown coefficients, whose behavior at large values of m is known [1]. We also have $W_m \sim m^{-1-\varkappa}\exp(-\pi md/2a)$ in the limit $m \to \infty$ [3]. Then

$$\sum_{m=1}^{\infty}W_m\frac{w}{h_{md}-w}=\sum_{m=1}^{M_w-1}W_m\frac{w}{h_{md}-w}+W\sum_{m=M_w}^{\infty}\frac{wm^{-1-\varkappa}}{h_{md}-w}.$$

Substituting (2) into equations analogous to those for properties IV and V of the function f(w), but which reflect the corresponding properties of h(w), we find a system of $M = M_u + M_v + M_w$ linear equations for the perturbing coefficients $\{U_m\}$, $\{V_m\}$, $\{W_m\}$, $\tilde U$, $\tilde V$, $\tilde W$. Solving this system for the cases of magnetic and electrical walls, and satisfying the normalization of the function h(w), we find the amplitudes of the field scattered by a dielectric inclusion of finite length.

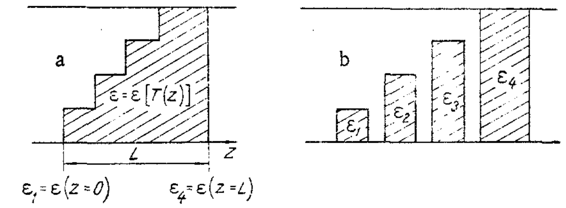

Fig. 2. a) Stepped dielectric wedge in waveguide in the four-step approximation; b) set of four separate elementary inhomogeneities, which are amenable to rigorous analysis.

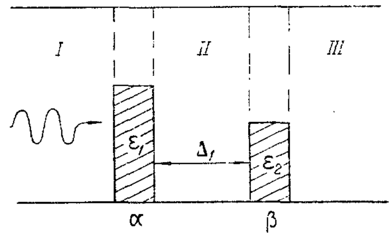

Fig. 3. Multiple scattering by a very simple set of elementary inhomogeneities.

In a study of the diffraction of a H_{0p} wave by a dielectric inclusion of finite length in a cylindrical waveguide we find a system of equations identical to (1), within the changes in definition specified in Part 3 of [1].

2. Calculation of Electrodynamic Characteristics of Inhomogeneities of Complex Shape in a Waveguide

A dielectric wedge is usually used as a "black-body" model in the microwave range. For an asymmetric wedge (tapering toward the broad wall), the diffraction problem has been solved by a rigorous method [5]. For the case of a symmetric wedge (with symmetric slopes toward both broad walls), which is frequently used, and for a wedge in a cylindrical waveguide, no study has been carried out. The method of generalized scattering matrices [6] can be used to solve the problem of the diffraction of electromagnetic waves by a symmetric dielectric obstacle simulating a "black body" in both circular and rectangular waveguides. This method is convenient in that the structure under study (Fig. 2a) is represented as a sequence of elements (Fig. 2b), each of which can be described by a generalized scattering matrix. Then the exact solution of the problem is written as a Neumann series containing matrices of infinite order, in an analysis of the multiple diffraction by the elementary inhomogeneities which are joined together.

Let us use the generalized scattering matrix for the case of the simplified structure in Fig. 3. We introduce the following notation: The amplitude of the p-th wave incident from region I is normalized to one. We denote the amplitude of the m-th reflected wave by $S_{11}^{\alpha}(p, m)$; the reflected mode has a zero phase at the front wall of inhomogeneity α. The amplitude of the m-th harmonic transmitted into region II is denoted by $S_{12}^{\alpha}(p, m)$, with a zero plane at the rear wall of inhomogeneity α. The quantities $S_{11}^{\alpha}(p, m)$ and $S_{12}^{\alpha}(p, m)$ are the elements of the scattering matrices S_{11}^{α} and S_{12}^{α}. The matrices representing the scattering by inhomogeneity β are S_{21}^{α}, S_{22}^{α}, S_{22}^{β} and S_{23}^{β}. We introduce the auxiliary matrix S_0, which describes the propagation of waves between inhomogeneities α and β. The elements of this matrix are

$$S_0(p, m)=\delta_{p,m}\exp(ih_m\Delta_1).$$

We associate the wave incident from region I with the infinite-dimensional vector $\vec{a}=\{a_p\}_{p=1}^{\infty}$, where the p-th column element is the amplitude of the p-th natural mode in the incident field. This field is scattered by inhomogeneities α and β. We describe the diffracted field by the following vectors: the vector reflected into region I is $S_{11}^{\alpha}\vec{a}$, while that transmitted into region II is $S_{12}^{\alpha}\vec{a}$. The vector reflected from inhomogeneity β into region II is $S_{21}^{\alpha}S_0S_{12}^{\alpha}\vec{a}$. The same field, transmitted into region I, is $S_{21}^{\alpha}S_0S_{22}^{\beta}S_0S_{12}^{\alpha}\vec{a}$. Summing the multiple scattering events in each of the regions, we find the following equation for the vectors of the field reflected into region I, $\vec{b}$, and that transmitted into region III, $\vec{c}$:

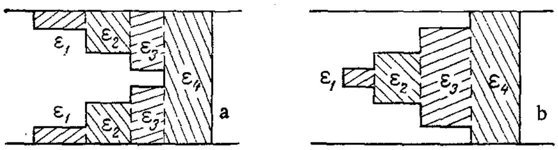

Fig. 4. Symmetric stepped dielectric wedges in waveguide. a) Convex; b) concave.

$$\vec{b} = S_{11}^{\alpha}\vec{a} + S_{21}^{\alpha}S_0S_{22}^{\beta}(E - S_0S_{22}^{\alpha}S_0S_{22}^{\beta})^{-1}S_0S_{12}^{\alpha}\vec{a}, \tag{3}$$

$$\vec{c} = S_{23}^{\beta}(E - S_0S_{22}^{\alpha}S_0S_{22}^{\beta})^{-1}S_0S_{12}^{\alpha}\vec{a}, \tag{4}$$

where E is the unit matrix.

Equations (3) and (4) give the exact solution of the problem.

By using this method along with the questions studied in Sec. 1, we can solve more complicated problems involving the scattering of electromagnetic waves by inhomogeneities of a broad class (Figs. 2a, 4a, and 4b), which represent a longitudinally inhomogeneous dielectric wedge. We allow the wedge profile along the waveguide to be arbitrary.

3. Determination of the Thermal Electromagnetic Fields

On the basis of the known amplitudes for the harmonics of the scattered field in the waveguide with the inhomogeneity, (3) and (4), and using the waveguide form of the electromagnetic fluctuation-dissipation theorem [2], we can solve the problem of finding the integral density of the fluctuation field of a waveguide structure heated to a temperature T:

$$\overline{|E_q^{\omega}|^2} = \frac{\hbar\omega}{\exp(\hbar\omega/T) - 1}\left[1 - \sum_{m=1}^{\infty}\frac{h_m}{h_q}\left(|b_{mq}|^2 + |c_{mq}|^2\right)\right].$$

By experimentally solving the inverse problem – that of determining the degree of heating of the inhomogeneity and of the adjacent region – we can use these structures as thermal microwave detectors from the plasma-heating region. As a result of these calculations we can offer practical recommendations on choosing the geometry in parameters of the structure used. The equations found for the reflection and transmission matrices are equally useful for working out recommendations for the development of concrete "black-body" models in the microwave range for the case of shielded structures. These questions are of much interest in the design of a matching waveguide transformer with an optimum profile. The solution method used in the present paper permits an efficient numerical application of the results.

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